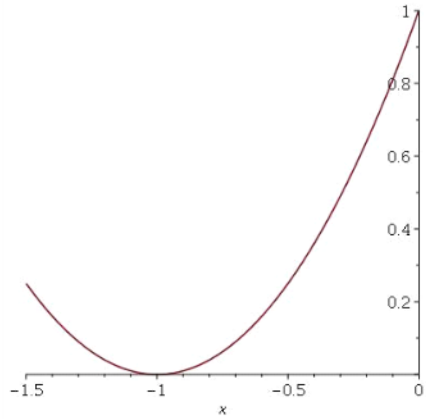
Lecture 3: 2.2, 2.3 Limits

September 14, 2016 5:42 PM

Limits

$$f(x) = \frac{x^3 + 3x^2 + 3x + 1}{x + 1}$$

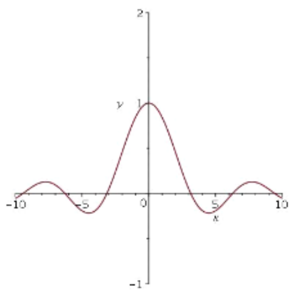


picture suggests that as f(x) approaches 0,
x --> -1
approaches

Intuitive definition of a limit: Suppose f(x) is defined when x is near the number a. Then we write:
$\lim_{x \to a} f(x) = L$
and say "the limit of f(x), as x approaches a, equals L" if we can make the values of f(x) arbitrarily close to to L by restricting x to be sufficiently close to a (but not equal to a).
(Table of values is NOT a valid answer to a limit)

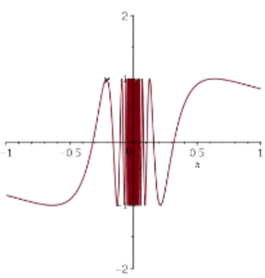
Ex

$\lim_{x\to 0} \frac{\sin x}{x}$	x	f(x)
as x approaches 0, f(x) approaches 1	0.5	0.95885
	0.25	0.98962
	0.1	0.98333
	0.05	0.99958
	0.025	0.99990



Ex

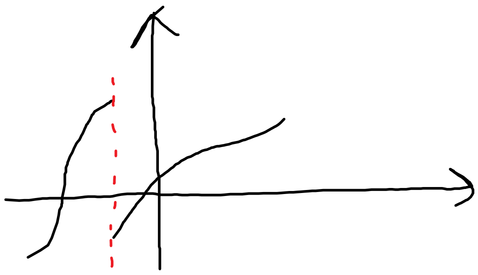
$\lim_{x\to 0} \sin \frac{1}{x}$	x	f(x)
no conclusion can be drawn. therefore, the limit does not exist	0.1	-0.544
	0.05	0.9129
	0.03	0.9405
	0.02	-0.2624



One-sided limits

We write $\lim_{x\to a^+} f(x) = L$ and say the right hand limit of f(x) as x-->a (*from the right*) is equal to L if we can make the values of f(x) arbitrarily close to L by taking x to be sufficiently close to a with x bigger than a.
Left hand limit: $\lim_{x\to a^-} f(x) = L$ (x approaches a *from the left*)

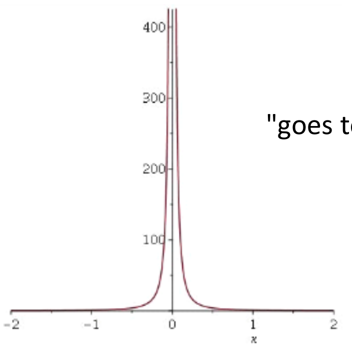
One sided limits are important because some functions are discontinuous:



The limit $\lim_{x \to a} f(x)$ exists at $x=a$ if $\lim_{x \to a^+} f(x) \in \mathbb{R}$ and $\lim_{x \to a^-} f(x) \in \mathbb{R}$ and $\lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x)$

Infinite limits

What is $\lim_{x \to 0} \frac{1}{x^2}$?



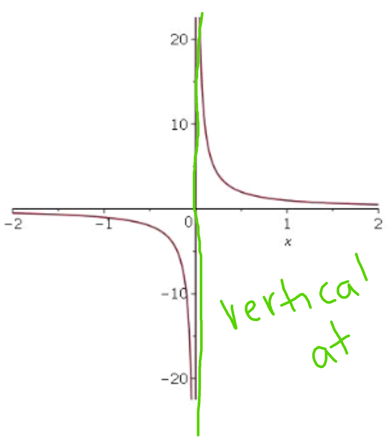
"goes to infinity"

x	y
0.5	4
0.25	16
0.125	64
0.001	10^6

as x approaches 0, y continues to get grow larger and larger - to infinity

But: infinity is not a real number, so we need a different notation (the infinity symbol).
Infinite limit: Let *f* be a function defined on both sides of *a*, except possibly at *a*, then $\lim_{x \to a} f(x) = \infty$ means that the values of *f(x)* can be made arbitrarily large by taking *x* sufficiently close to *a*, but not equal to *a*.
We use $\lim_{x \to a} f(x) = -\infty$ if we can make f(x) arbitrarily small by taking *x* suffieciently close *to a*.

<u>Ex</u> $f(x) = \frac{1}{x}$



left hand limit ≠ right hand limit (-∞ and ∞)

vertical asymptote at $x=0$

Limit rules

$\lim_{x \to a} x = a$

$$\lim_{x\to a} x = a$$

$$\lim_{x\to a} (f(x) \pm g(x)) = \lim_{x\to a} f(x) \pm \lim_{x\to a} g(x)$$

$$\lim_{x\to a} (c\, f(x)) = c \lim_{x\to a} f(x)$$

$$\lim_{x\to a} (f(x)\, g(x)) = \lim_{x\to a} f(x) \lim_{x\to a} g(x)$$

$$\lim_{x\to a} \left(\frac{f(x)}{g(x)}\right) = \frac{\lim f(x)}{\lim g(x)}, \quad \lim_{x\to a} g(x) \neq 0$$

$$\lim_{x\to a} c = c, \quad c \in \mathbb{R}$$

Examples

1. $\lim_{x\to 5}(2x^2 - 3x + 4)$

$= \lim_{x\to 5}(2x^2) - \lim_{x\to 5}(3x) + \lim_{x\to 5} 4$

$= 2 * \lim_{x\to 5}(x^2) - 3 * \lim_{x\to 5}(x) + 4$

$= 2 * 5^2 - 3.5 + 4$

$= 39$

2. $\lim_{x\to 1}(\frac{x^2-1}{x-1})$

$= \lim_{x\to 1} \frac{(x+1)\cancel{(x-1)}}{\cancel{x-1}}$

$= \lim_{x\to 1}(x+1)$

$= 2$

3.

find common denom

$\lim_{x\to 3}(\frac{\frac{1}{x} - \frac{1}{3}}{x-3})$ turn into only one fraction line

$= \lim_{x\to 3}(\left(\frac{3-x}{3x}\right) * \frac{1}{x-3})$

$= \lim_{x\to 3}(\frac{3-x}{3x(x-3)})$ multiply by (-1)

$= -\lim_{x\to 3}(\frac{\cancel{-3+x}}{3x\cancel{(x-3)}})$

$= -\lim_{x\to 3}(\frac{1}{3x})$

$= -\frac{1}{9}$ dont forget (-)